# On inelastic seismic response of asymmetric single-story structures

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## ABSTRACT

A parametric study of the inelastic seismic response of idealised single-story asymmetric buildings has been performed. All models included load resisting elements in both orthogonal directions. The main objective was to compare the results obtained by uni- and bi-directional ground motion. Preliminary results, based on a very limited number of test models, indicate that fair estimates of the relevant maximum response quantities can be obtained by performing two independent uni-directional analyses and combining the results by the SRSS rule. This conclusion, if confirmed in future studies, enables a simple extension of simplified nonlinear analyses, based on pushover analysis, to asymmetric building structures.

### INTRODUCTION

"Progress in understanding the seismic response behaviour of asymmetric building systems has been rather slow, considering the research efforts that have been devoted to this subject during the last four decades. This is manifested in the relative scarcity of conclusions of general validity". These opening sentences in a paper by Chandler et al (1996) are still relevant in 1999. The main reason seems to be a very large number of parameters, which influence the nonlinear response. In individual research projects, that are not co-ordinated world-wide, it is practically impossible to consider all parameters.

A large number of papers has been published on inelastic seismic response of asymmetric buildings. A bibliography of publications up to the end of 1994 was prepared by Rutenberg et al (1995). Later on, important contributions have been made by De la Llera and Chopra (1995), Bertero (1995), Tso and Wong (1995), Paulay (1996), and De Stefano et al (1998), inter alia. Several relevant papers were presented at a specialized workshop (Ramasco and Rutenberg, 1996). For the topic, discussed in this paper, is especially interesting the work by Correnza and coauthors (1994 and 1997). In the first paper it is concluded that "models incorporating the transverse elements but analyzed under uni-directional lateral loading may underestimate by up to 100% the torsional effects in such systems, but are reasonably accurate for medium-and long-period structures". One of the conclusions of the second paper is that "both perpendicular horizontal earthquake components must be considered when using models with transversely-oriented elements".

In this paper, a part of the research performed at the University of Ljubljana, aimed to better understand the inelastic seismic response of asymmetric structures, is presented. We are trying to get some information on the difference between the response of structures subjected to uni- and bi-directional horizontal excitation. We would like to contribute to the solution of a problem that is very important for the future development of practical design procedures: How to perform a simplified nonlinear analysis of asymmetric building structures? Such analyses, which are rapidly gaining popularity, combine a nonlinear static (pushover) analysis with response spectrum approach. Simplified nonlinear methods have been developed for planar structures and their applicability has to be extended to asymmetric structures that experience torsional rotations. Here the problem arises how to combine the influence of the two horizontal excitations. The answer of this question is one of the final objectives of the research reported in this paper.

## METHODOLOGY AND MATHEMATICAL MODELS

A parametric study of highly idealised single-story building structures was performed (in parallel, the response of multistory buildings was also studied. Some results are summarised in the companion paper by Kilar and Fajfar, 1999). When designing the mathematical model, the following consideration were made:

Several researchers have emphasised the utmost importance of the transverse structural elements (i.e. the load-resisting elements oriented perpendicular to the assumed horizontal direction of earthquake loading). Consequently, the authors believe that, in principle, only models with elements oriented in two directions should be used for realistic simulation of the actual behaviour, even if some studies have demonstrated, for a restricted range of input parameters, that appropriate results can be obtained also with elements only in the direction of loading.

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The distribution of strength in the majority of the models investigated so far has been determined according to a specific code. We claim that a strength distribution strictly following a code can never be attained in a real structure. Because of this reason, and because of our ambition to explore the inelastic response in a more general way, independently of specific codes, we decided to investigate some limit cases regarding strength distribution.

There is an important difference between the seismic response of torsionally stiff (first mode is translational) and torsionally flexible (first mode is torsional) buildings. In the case of torsionally flexible buildings, the dynamic torsional influence may be very important. Usually, the maximum displacement at the stiff edge of the building is larger than that at the flexible edge. Such behaviour is qualitatively different from that observed in torsionally stiff buildings and from that obtained in the case of static loading in mass centre. There is a general agreement that it is preferable not to use torsionally flexible buildings. We are investigating also such structures. In this paper, however, the discussion will be restricted to torsionally stiff buildings.

Like the majority of researchers, we used a model with three elements in each of two orthogonal directions (Figure 1). In the great majority of cases, the model was asymmetric with respect to both horizontal axes. This is a difference compared to usual models, which are symmetric regarding the transversal axis. Some researchers have observed that the short-period structures are more sensitive to the different modelling options than medium- and long-period structures. Based on this observation, we started our investigation with short-period structures. In subsequent analyses we will study also more flexible structures.

So far, we have been able to partially investigate only a restricted number of parameters:

- type of eccentricity (mass, stiffness and strength eccentricity),
- magnitude of eccentricity, and
- characteristics of transverse elements.

For each structural model two variants were analysed. The first one, with limited strengths, responded in inelastic range. The response of the corresponding model with infinite strength was linear elastic.



Figure 1: Plan of symmetric single-story building (Stiff- strong and flexible-weak sides for asymmetric structures are indicated).

Data for the basic symmetric model (a / b = 1.5):

stiffness K, strength  $F_v = 0.256$  Mg, M is total mass

elements  $Y_1$ ,  $Y_2$ ,  $Y_3$ stiffness k = K/3, strength  $f_y = F_y/3$ yield displacement  $u_y = 1.02$  cm

elements  $X_1$ ,  $X_2$ ,  $X_3$ stiffness 1.778 k = 0.593 K, strength 1.5 f<sub>y</sub>= 0.5 F<sub>y</sub> yield displacement  $u_y$ = 0.86 cm

periods:  $T_X = 0.3$  s,  $T_Y = 0.4$  s,  $T_{\infty} = 0.254$  s



Figure 2: Force - displacement relations for Y-elements in asymmetric structures. The same ratios apply for X-elements.

The basic symmetric model is shown in Figure 1. Six lateral load-resisting elements are connected with a slab, which is rigid in horizontal plan and has no stiffness in the vertical direction. Mass is uniformly distributed at the slab level. The hysteretic behaviour of all elements is ideal elasto-plastic with 5% post-yield stiffness. The static force-displacement relations for different elements, representing envelopes of the hysteretic loops, are shown in Figure 2. In the majority of models, the eccentricity in both horizontal directions amounts to 10% of the corresponding dimension in plan. Mass eccentricity is produced by shifting the mass centre from the geometrical centre. Stiffness and or strength eccentricity is

obtained by changing the characteristics of elements (see Figure 2). In the SR model, a linear relation between stiffness and strength is assumed. Consequently, both stiffness and strength eccentricities are the same. In the SRa model, it is more realistically assumed that the changes of strength are smaller than the changes of stiffness. Mass eccentricities in models MY5, MY0 and M30 are different as in model M. Model MY0 is symmetric regarding X-axis. In the MXY model the characteristics of the elements in the transverse direction are changed in order to obtain equal uncoupled period of translational motion in two horizontal directions. In the model MG the geometry of the plan was changed from rectangular to square. The structural elements are the same as in the model MXY. An overview of the investigated models is given below. Only those characteristics which are different from the basic symmetric structure are given. Normalised eccentricities are defined as the ratios of mass, stiffness and/or strength eccentricity and the corresponding dimension in the plan (a or b):

Model M	mass ecc.	$e_{MX} = -0.1, e_{MY} = 0.1$
Model S	stiffness ecc.	$e_{SX} = 0.1, e_{SY} = -0.1$ , see Fig. 2
Model R	strength ecc.	$e_{RX} = 0.1, e_{RY} = -0.1$ , see Fig. 2
Model SR	strength and stiffness ecc.	$e_{SX} = e_{RX} = 0.1$ , $e_{SY} = e_{RY} = -0.1$ , see Fig. 2
Model SRa	strength and stiffness ecc.	$e_{SX} = 0.1$ , $e_{RX} = 0.084$ , $e_{SY} = -0.1$ , $e_{RY} = -0.084$ , see Fig. 2
Model MY5	mass ecc.	$e_{MX} = -0.1, e_{MY} = 0.05$
Model MY0	mass ecc.	$e_{MX} = -0.1, e_{MY} = 0$
Model M30	mass ecc.	$e_{MX} = -0.3, e_{MY} = 0.3$
Model MXY	mass ecc.	$e_{MX} = -0.1, e_{MY} = 0.1, T_Y = T_X$
Model MG	mass ecc.	$e_{MX} = -0.1, e_{MY} = 0.1, a = b$

With the exception of the models MXY and MG, the periods of uncoupled vibrations are equal to the periods of the basic symmetric structure (i.e.  $T_x = 0.3$ ,  $T_y = 0.4$  s). In models MXY and MG, the uncoupled periods of vibration in X- and Y-direction are equal ( $T_x = T_y = 0.3$  s). This change is made by using in Y-direction the same elements as in X-direction. The periods of uncoupled torsional vibration are 0.254 s, 0.262 s, 0.212 s and 0.180 s for M (same for R, MY5, MY0, M30), S (same for SR, SRa), MXY and MG models, respectively.

Two horizontal components of eight strong motion records (Sylmar and Newhall from Northridge 1994, Kobe J.M.A. from Kobe 1995, El Centro 1940, Petrovac, Ulcinj 1, Ulcinj 2 and Bar from Montenegro 1979) were used for time-history analyses. Each pair of accelerograms was applied twice (in the second run the X- and Y-directions were interchanged). So, sixteen time-histories were computed for each system. The fundamental periods of the investigated structures are in the short-period range of spectra. Consequently, the records were normalised regarding peak ground acceleration. After scaling, for each record the maximum value of ground acceleration in horizontal plane (considering time-histories of both components) was equal to 0.4 g. Normalised acceleration spectra are shown in Figure 3. The stronger components (i.e. for each accelerogram the component with larger peak ground acceleration) were grouped together. These are N-S components of all records except Ulcinj 2, where the E-W component is stronger.





#### RESULTS

Main results of the parametric study are summarised in Tables 1 and 2. Results of inelastic and elastic analyses are given. In Table 1 the influence of asymmetry is shown. Values in Table 1 were calculated for each model as follows. For each of 16 bi-directional ground excitations, maximum response in terms of displacements at all four sides of the building and at the geometrical centre were determined. Then, the mean values were calculated. In the table the ratios between the mean values of asymmetric model and of the basic symmetric model are given. In the case of the symmetric model (Figure 1), the absolute values of inelastic displacements are 2.06 cm and 4.62 cm in X- and Y-direction. respectively. The corresponding ductilities amount to 2.40 and 4.53, respectively. In the case of unlimited elastic behaviour, the displacements amount to 1.92 cm and 3.29 cm in X- and Y-direction, respectively. For the models MXY and MG, the displacements and ductilities in Y-direction are equal to those in X-direction.

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			inel	astic		elastic							
model	X <sub>flex</sub>	X <sub>stiff</sub>	Y <sub>flex</sub>	Y <sub>stiff</sub>	X <sub>c</sub>	Y <sub>c</sub>	Xflex	X <sub>stiff</sub>	Yflex	Ystiff	X <sub>c</sub>	Ye	
М	1.45	1.03	1.15	0.85	1.07	0.92	1.28	0.84	1.25	0.91	0.94	0.99	
S	1.28	1.04	1.08	0.99	1.06	1.00	1.38	0.84	1.36	0.91	1.02	1.02	
R	1.27	1.09	1.17	0.82	1.04	0.93	1.00	1.00	1.00	1.00	1.00	1.00	
SR	1.49	0.97	1.20	0.82	1.07	0.94	1.38	0.84	1.36	0.91	1.02	1.02	
SRa	1.31	1.05	1.11	0.93	1.10	0.96	1.38	0.84	1.36	0.91	1.02	1.02	
MY5	1.33	1.07	1.18	0.78	1.01	0.93	1.19	0.94	1.23	0.82	0.97	0.98	
MY0	1.26	1.15	1.19	0.78	1.03	0.95	1.06	1.05	1.22	0.76	1.00	0.97	
M30	1.73	1.08	1.23	0.65	1.08	0.74	1.64	0.97	1.58	0.73	0.96	1.00	
MXY	1.18	1.01	1.39	0.72	1.03	0.95	1.17	0.90	1.35	0.77	0.98	0.97	
MG	1.31	0.95	1.31	0.95	1.03	1.03	1.22	0.82	1.22	0.82	0.98	0.98	

Table 1. Ratio of displacements (asymmetric / symmetric, bi-directional input).

	inelastic								elastic						
model	$X_{flex}$	X <sub>stiff</sub>	Y <sub>flex</sub>	Ystiff	Xc	Y <sub>c</sub>	φ	$X_{flex}$	X <sub>stiff</sub>	Y <sub>flex</sub>	Ystiff	X <sub>c</sub>	Yc	φ	
М	0.84	0.85	0.98	1.01	0.96	1.07	0.49	0.94	0.90	0.98	0.81	1.01	0.98	0.67	
S	0.98	1.00	0.99	1.04	1.05	1.03	0.60	0.92	0.91	0.95	0.83	0.96	0.99	0.73	
R	0.88	0.80	0.98	1.03	0.92	1.05	0.62	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
SR	0.86	0.91	0.96	1.02	0.99	1.04	0.61	0.92	0.91	0.95	0.83	0.96	0.99	0.73	
SRa	0.97	0.92	1.00	1.06	1.00	1.08	0.58	0.92	0.91	0.95	0.83	0.96	0.99	0.73	
MY5	0.85	0.85	0.95	1.11	0.99	1.06	0.56	0.95	0.90	0.99	0.92	1.01	0.99	0.84	
MY0	0.78	0.86	0.94	1.09	0.96	1.03	0.61	0.94	0.95	1.00	1.00	1.00	1.00	1.00	
M30	0.74	0.67	1.00	0.73	0.85	0.98	0.85	0.71	0.69	0.93	0.84	0.88	0.84	0.87	
MXY	0.97	0.95	0.98	0.89	1.01	0.99	0.81	0.94	0.92	0.95	0.85	0.98	0.97	0.80	
MG	0.92	0.89	0.92	0.89	0.97	0.97	0.51	0.96	0.95	0.96	0.95	0.98	0.98	0.70	

Table 2a. Ratio of displacements and rotations obtained by uni- and bi-directional input ground motion

Tuble 2b. Ratio of displacements and rotations obtained by uni- and bi-directional input ground motion (comb.values).

	inelastic								elastic						
model	X <sub>flex</sub>	X <sub>stiff</sub>	Y <sub>flex</sub>	Ystiff	X <sub>c</sub>	Y <sub>c</sub>	φ	$X_{flex}$	X <sub>stiff</sub>	Y <sub>flex</sub>	Ystiff	$X_c$	Y <sub>c</sub>	φ	
М	0.88	0.91	0.99	1.06	0.97	1.07	0.65	1.00	0.98	0.99	0.94	1.02	0.99	0.94	
S	1.00	1.01	1.00	1.06	1.06	1.04	0.73	0.99	0.97	0.98	0.95	0.99	1.00	0.99	
R	0.92	0.86	0.99	1.05	0.92	1.05	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
SR	0.93	0.96	0.98	1.06	1.00	1.05	0.79	0.99	0.97	0.98	0.95	0.99	1.00	0.99	
SRa	1.00	0.93	1.01	1.09	1.00	1.09	0.74	0.99	0.97	0.98	0.95	0.99	1.00	0.99	
MY5	0.89	0.90	0.96	1.13	0.99	1.06	0.65	1.00	0.97	1.00	0.96	1.01	1.00	0.98	
MY0	0.83	0.90	0.94	1.09	0.96	1.03	0.61	0.99	1.00	1.00	1.00	1.00	1.00	1.00	
M30	0.95	0.91	1.05	0.90	0.91	1.06	1.00	0.98	0.88	1.03	0.99	1.03	0.99	1.01	
MXY	1.02	1.01	1.00	0.98	1.03	1.01	0.90	0.99	0.99	0.99	0.94	1.01	1.00	0.96	
MG	0.95	0.94	0.95	0.94	0.98	0.98	0.73	0.99	1.01	0.99	1.01	1.01	1.01	0.99	

In Table 2 the results obtained for uni-and bi-directional excitation are compared. The computational procedure was similar as for Table 1. The uni-directional results in Table 2a correspond to the larger of the two values obtained by performing two separate analyses with uni-directional ground motion in X- and Y-direction, respectively. In Table 2b both values, obtained by separate analyses with uni-directional input, are combined according to the SRSS rule, i.e.  $R = \sqrt{(R_x^2 + R_y^2)}$ , where R denotes any response quantity.  $R_x$  and  $R_y$  are values of the quantity R obtained by applying ground excitation only in X- and only in Y-direction, respectively. The values of the coefficient of variation, indicating the scatter of results obtained for 16 time-histories, vary from about 0.1 to about 0.4 with an average of about 0.3.

From Table 1 an interesting trend can be noticed. The torsional influence is almost in all cases larger in X- than in Ydirection. Small increase of displacements can be observed even on the stiff side. This observation applies only for inelastic response. The observed phenomenon might be a consequence of the fact that the elements in Y-direction (i.e. transverse regarding to X) experience larger inelastic deformations.

Tables 2 demonstrate that, in the majority of cases, a fair (slightly underestimated) estimate of displacements can be obtained from uni-directional ground motion. It is interesting that the best correlation is achieved for the most critical part of the structure, i.e. the flexible side in Y-direction. In general, the estimates based on uni-directional ground motions can be improved if the SRSS rule is applied (Table 2b). This rule has been widely used for elastic structures. In our study it provided excellent results (see also right part of Table 2b). The results for inelastic structures are not as favourable, but still enough accurate for most practical purposes.



time (s) Figure 4: Time-histories of plastic mechanisms and input ground motions.



Figure 5: Snapshots of displaced building (displacements and torsional rotations are magnified by a factor of 100).

In the case of torsional rotation of inelastic structures, uni-directional results largely underestimate the bi-directional ones. However, this fact has in majority of cases no practical consequences, because maximum rotations generally do not occur at the same time as maximum displacements.

In order to better understand the inelastic torsional behaviour, a more detailed presentation of the response of the SR model to Petrovac ground motion (N-S component in X-direction and E-W component in Y-direction) is given in Figure 4. Time-histories of plastic mechanisms are shown for uni-directional as well as for bi-directional input. Indicated are translational mechanisms in X- and Y-direction, and torsional mechanism. It can be seen that plastic mechanisms occur a limited number of times and they last only a short time. Consequently, it is reasonable to expect, that in the large majority of cases, many transverse elements are elastic at the moment when the elements in the direction of larger load yield. In such cases, the major part of torsion is resisted by elastic transverse elements.

#### CONCLUSIONS

Based on the very limited number of test models some preliminary conclusions can be drawn. In general, but not always, uni-directional input slightly underestimates the response obtained by bi-directional input. The agreement can be improved, and results acceptable for most practical purposes can be obtained, by using the SRSS rule for combining the results obtained by two independent uni-directional analyses, one for each of two horizontal directions. In all cases, the transverse elements must be included. Further investigations are needed. If the above observations are confirmed, it will be possible to extend the applicability of the simplified nonlinear analysis procedure to asymmetric structures. The analysis will consist of two independent pushover analyses in two horizontal directions.

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#### REFERENCES

Bertero, R.D. 1995. Inelastic torsion for preliminary seismic design. Journal of Structural Engineering. ASCE, 121, 1183-1189.

Chandler, A.M., Duan, X. N., and Rutenberg, A. 1996. Seismic torsional response: assumptions. controversies and research progress. European Earthquake Engineering, 10, 37-51.

Correnza, J.K., Hutchinson, G.L., and Chandler, A.M., 1994. Effect of transverse load-resisting elements on inelastic earthquake response of eccentric-plan buildings. Earthquake Engineering and Structural Dynamics, 23, 75-89.

Correnza, J.K., and Hutchinson, G.L., 1997. Inelastic response of code-designed eccentric structures subject to bidirectional loading. Structural Engineering and Mechanics, 5, 51-58.

De La Llera, J.C. and Chopra, A.K. 1995. Understanding the inelastic seismic behaviour of asymmetric-plan buildings. Earthquake Engineering and Structural Dynamics, 24, 549-572.

De Stefano, M., Faella, G., and Ramasco, R. 1998. Inelastic seismic response of one-way plan-asymmetric systems under bidirectional ground motions. Earthquake Engineering and Structural Dynamics, 27, 363-376.

Kilar, V. and Fajfar, P. 1999. Inelastic seismic response of an asymmetric multi-story RC frame building. 8th Canadian Conf. Earthquake Engineering, Vancouver.

Paulay, T. 1996. Seismic design for torsional response of ductile buildings. Bull. of the New Zealand Nat. Soc. for Earthquake Engineering, 29, 178-197.

Ramasco, R., and Rutenberg, A., editors, 1996. Proceedings of the European Workshop on the seismic behaviour of asymmetric and setback structures. Anacapri, Italy.

Rutenberg, A., Chandler, A.M., Duan, X.N., and Correnza, J.K. 1995. Nonlinear seismic response of asymmetric structures: Bibliography. National Building Research Institute, Haifa, Israel.

Tso,W.K.,and Wong,C.M., 1995. Seismic displacements of torsionally unbalanced buildings. Earthquake Engineering and Structural Dynamics, 24, 1371-1387.